# Lab: Recursion and the 8 Queens Puzzle

This document defines the **in-class exercises** (lab) for the ["Algortihms" course @ Software University](https://softuni.bg/trainings/1194/Algorithms-September-2015).

In this lab we will implement a recursive algorithm to solve the **“8 Queens” puzzle**. Our goal is to write a program to **find all possible placements of 8 chess queens** on a chessboard, so that no two queens can attack each other (by horizontal, vertical or diagonals).

## Learn about the “8 Queens” Puzzle

Learn about the “8 Queens” puzzle, e.g. from Wikipedia: <http://en.wikipedia.org/wiki/Eight_queens_puzzle>.

## Define a Data Structure to Hold the Chessboard

First, let’s define a data structure to hold the **chessboard**. It should consist of 8 x 8 cells, each either occupied by some queen or empty. Let’s also define the size of the chessboard as a constant:



## Define a Data Structure to Hold the Attacked Positions

We need to **hold the attacked positions** in some data structure. At each moment during the work of the program we need to know **whether certain** **position** **{row, col} is under attack** by some queen or not.

There are many ways to **preserve the attacked positions**:

* By keeping **all currently placed queens** and checking whether the new position conflicts with some of them.
* By keeping an int[,] **matrix of all attacked positions** and checking the new position directly in it. This will be complex to maintain because the matrix should change many positions after each queen place / removal.
* By keeping **sets of all attacked rows, columns and diagonals**. Let’s try this idea:

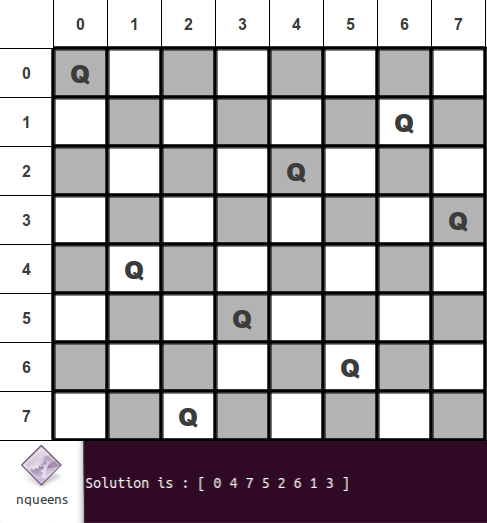


The above definitions have the following assumptions:

* **Rows** are 8, numbered from 0 to 7.
* **Columns** are 8, numbered from 0 to 7.
* The **left diagonals** are 15, numbered from -7 to 7. We can use the following formula to calculate the left diagonal number by row and column: leftDiag = col - row.
* The **right diagonals** are 15, numbered from 0 to 14 by the formula: rightDiag = col + row.

Let’s take as an **example** the following chessboard with 8 queens placed on it at the following positions:

* {0, 0}; {1, 6}; {2, 4}; {3, 7}; {4, 1}; {5, 3}; {6, 5}; {7, 2}



Following the definitions above for our example the **queen {4, 1}** occupies the **row 4**, **column 1**, **left diagonal -3** and **right diagonal 5**.

## Write the Backtracking Algorithm

Now, it is time to write the recursive **backtracking algorithm** for placing the 8 queens.

The algorithm starts from row 0 and tries to place a queen at some column at row 0. On success, it tries to place the next queen at row 1, then the next queen at row 2, etc. until the last row is passed. The code for putting the next queen at certain row might look like this:



Initially, we invoke this method from row 0:



## Check if a Position is Free

Now, let’s write **the code to check whether certain position is free**. A position is free when it is not under attack by any other queen. This means that if some of the rows, columns or diagonals is already occupied by some other queen, the position is occupied. Otherwise it is free. A sample code might look like this:



Recall that col-row is the number of the left diagonal and row+col is the number of the right diagonal.

## Mark / Unmark Attacked Positions

After a queen is placed, we need to **mark as occupied all rows, columns and diagonals** that it can attack:



In the opposite way, we will need a method to mark as free all rows, columns and diagonals that can be attacked by certain queen position. Write it yourself:



## Print Solutions

When a solution is found, it should be printed at the console. First, introduce a solutions counter to simplify checking whether the found solutions are correct:



Next, pass through all rows and through all columns at each row and **print the chessboard cells**:



## Testing the Code

The “8 queens” puzzle has **92 distinct solutions**. Check whether your code generates and prints all of them correctly. The solutionsFound counter will help you to check the number of solutions. Below are the 92 distinct solutions:

|  |
| --- |
|  |
| http://www.datagenetics.com/blog/august42012/s01.png | http://www.datagenetics.com/blog/august42012/s02.png | http://www.datagenetics.com/blog/august42012/s03.png | http://www.datagenetics.com/blog/august42012/s04.png | http://www.datagenetics.com/blog/august42012/s05.png | http://www.datagenetics.com/blog/august42012/s06.png | http://www.datagenetics.com/blog/august42012/s07.png | http://www.datagenetics.com/blog/august42012/s08.png |
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## Optimize the Solution

Now we can optimize our code:

* Remove the attackedRows set. It is not needed because all queens are placed consecutively at rows 0…7.
* Try to use bool[] array for attackedColumns, attackedLeftDiagonals and attackedRightDiagonals instead of sets. Note that arrays are indexed from 0 to their size and cannot hold negative indexes.

## \* Permutation Based Solution

Try to implement the more-efficient **permutation-based solution** of the “8 Queens” puzzle. Look at this code to catch the idea: <http://introcs.cs.princeton.edu/java/23recursion/Queens.java.html>.